Find the length of the curve $y=\frac{1}{3}(x-3)\sqrt{x}$ over $4\le x\le 16$. **SCORE: _____ / 25 PTS**

NOTE: Your final answer must be a number.

$y=\frac{1}{3}x^{\frac{3}{2}}-x^{\frac{1}{2}}$

$\frac{dy}{dx}=\underbrace{\frac{1}{2}x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}}$ 4

2 $\underbrace{\int_4^{16}\sqrt{1+\left(\frac{1}{2}x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}\right)^2}\,dx}$ 2 ← MUST BE PRESENT IN ALL $\int$

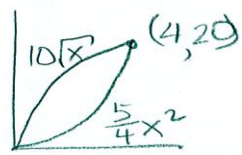
$=\int_4^{16}\sqrt{\overset{3}{1+\frac{1}{4}x-\frac{1}{2}+\frac{1}{4}x^{-1}}}\,dx$

$=\int_4^{16}\sqrt{\underbrace{\frac{1}{4}x+\frac{1}{2}+\frac{1}{4}x^{-1}}_{3}}\,dx$

$=\int_4^{16}\left(\underbrace{\frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}}_{3}\right)dx$

$=\underbrace{\left(\frac{1}{3}x^{\frac{3}{2}}+x^{\frac{1}{2}}\right)}_{3}\Big|_4^{16}=\underbrace{\frac{1}{3}(64-8)+(4-2)}^{3}=\frac{56}{3}+2=\underbrace{\frac{62}{3}}^{2}$

Find the centroid (center of mass) of the region bounded by $y=10\sqrt{x}$ and $y=\frac{5}{4}x^2$. **SCORE: _____ / 25 PTS**



NOTE: Your final answer must be numeric.

$\int_0^4\left(10x^{\frac{1}{2}}-\frac{5}{4}x^2\right)dx=\underbrace{\left(\frac{20}{3}x^{\frac{3}{2}}-\frac{5}{12}x^3\right)}\Big|_0^4$ 3 $=\frac{20}{3}\cdot 8-\frac{5}{\cancel{12}\,3}\cdot\cancel{64}\,16=\frac{160-80}{3}=\underbrace{\frac{80}{3}}_{2}$

$\underbrace{\int_0^4 x\left(10x^{\frac{1}{2}}-\frac{5}{4}x^2\right)dx}_{3}=\int_0^4\left(10x^{\frac{3}{2}}-\frac{5}{4}x^3\right)dx=\underbrace{\left(4x^{\frac{5}{2}}-\frac{5}{16}x^4\right)}_{3}\Big|_0^4$

$=4\cdot 32-\frac{5}{\cancel{16}}\cdot\cancel{4^4}\,16$

$=128-80=\underbrace{48}_{2}$

3

$\underbrace{\frac{1}{2}\int_0^4\left(\left(10x^{\frac{1}{2}}\right)^2-\left(\frac{5}{4}x^2\right)^2\right)dx}=\frac{1}{2}\int_0^4\left(100x-\frac{25}{16}x^4\right)dx$

$=$ 3 $\underbrace{\frac{1}{2}\left(50x^2-\frac{5}{16}x^5\right)}\Big|_0^4$

-1 IF MISSING ANY dx (1 DEDUCTION ONLY)

$=\frac{1}{2}\left(50\cdot 16-\frac{5}{\cancel{16}}\cdot\cancel{4^5}\,4^3\right)$

$=\frac{1}{2}(800-320)=\underbrace{240}$ 2

$\frac{1}{\frac{80}{3}}(48,240)=\frac{3}{80}(48,240)=$ 2 $\left(\underbrace{\frac{9}{5}},\underbrace{9}\right)$ 2

A certain stop light is programmed to take traffic conditions into consideration. A driver who pulls up to the light when it is red will have to wait between 1 and 27 seconds. Let X be the amount of time (in seconds) that a random driver must wait for a red light to turn. Find the average (mean) time that a random driver has to wait if the probability density function of X has the form **SCORE: _____ / 25 PTS**

$$f(x) = \begin{cases} \frac{k}{\sqrt[3]{x}}, & x \in [1, 27] \\ 0, & x \notin [1, 27] \end{cases}$$

NOTE: Your final answer must be a number.

$\int_1^{27} kx^{-\frac{1}{3}} dx = 1$ (4)

(4) $\frac{3}{2}kx^{\frac{2}{3}}\Big|_1^{27} = 1$

$\frac{3}{2}k(9-1) = 1$

(4) $k = \frac{1}{12}$

$\int_1^{27} \frac{1}{12} x \cdot x^{-\frac{1}{3}} dx$ (5)

$= \int_1^{27} \frac{1}{12} x^{\frac{2}{3}} dx$

$= \frac{1}{12} \frac{3}{5} x^{\frac{5}{3}}\Big|_1^{27}$ (4)

$= \frac{1}{20}(243-1)$

$= \frac{242}{20}$

$= \frac{121}{10}$ (4) $= 12.1$ SECONDS

Consider the region bounded by $y = 2x - 10$, $y = \frac{1}{2}x - 1$ and $x = 0$. **SCORE: _____ / 35 PTS**

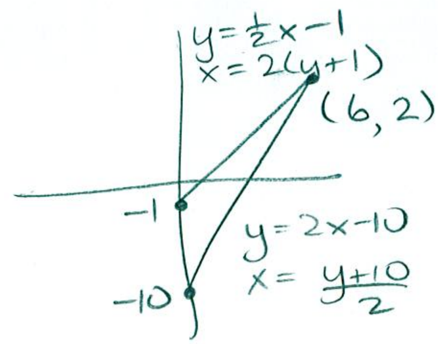
[a] Write, **BUT DO NOT EVALUATE**, a dy integral (or sum of integrals) for the volume if the region is revolved around the line $x = -3$.

20

$\pi \int_{-10}^{-1} \left[\left(\frac{y+10}{2}+3\right)^2 - (3)^2\right] dy$

$+ \pi \int_{-1}^{2} \left[\left(\frac{y+10}{2}+3\right)^2 - (2(y+1)+3)^2\right] dy$

(3) (3) (2) (1) (2) (3) (2) (3) (1)



[b] Write, **BUT DO NOT EVALUATE**, a **single** integral for the volume if the region is revolved around the line $x = 8$.

15

$2\pi \int_0^6 (8-x)\left(\frac{1}{2}x-1-(2x-10)\right) dx$

(2) (3) (3) (5) (2)

A solid of revolution has volume $\int_1^4 2\pi(e^y + y)(6-y)\,dy$. Sketch the region and the axis of revolution. **SCORE: _____ / 15 PTS**

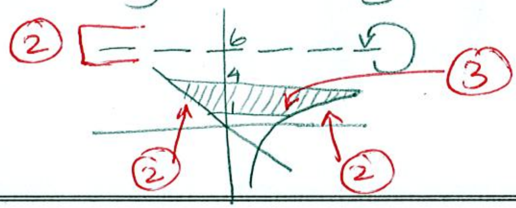
Do not use the *x*– nor *y*–axes as boundaries nor the axis of revolution.

SHELL METHOD, HORIZONTAL CUT // HORIZONTAL AXIS OF REVOLUTION

$r = 6 - y$ AXIS $\underline{y = 6}$ (2)

$h = e^y + y = e^y - (-y)$ BOUNDARIES $\underline{x = e^y}$ (1) $\rightarrow \underline{y = \ln x}$ (1)

$\underline{x = -y}$ (1) $\rightarrow \underline{y = -x}$ (1)



Find the area of the region between the curves $y = \cos x$ and $y = \sin 2x$ over $0 \le x \le \frac{\pi}{2}$. **SCORE: _____ / 25 PTS**

NOTE: Your final answer must be a number.

$\cos x = \sin 2x$

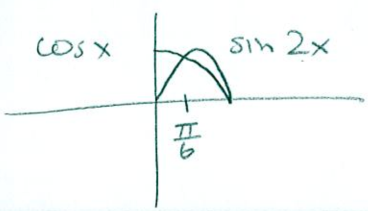
$\cos x = 2\sin x \cos x$

$0 = 2\sin x\cos x - \cos x$

$= (2\sin x - 1)\cos x$

$2\sin x - 1 = 0 \rightarrow \sin x = \frac{1}{2} \rightarrow x = \frac{\pi}{6}$

$\cos x = 0 \rightarrow x = \frac{\pi}{2}$



(4) $\int_0^{\frac{\pi}{6}} (\cos x - \sin 2x)\,dx$ (2) (1)

(4) $+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x)\,dx$ (2) (1)

$= \left(\sin x + \frac{1}{2}\cos 2x\right)\Big|_0^{\frac{\pi}{6}}$ (4)

$+ \left(-\frac{1}{2}\cos 2x - \sin x\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ (1)

$= \left[\left(\frac{1}{2} + \frac{1}{4}\right) - \left(0 + \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - 1\right) - \left(-\frac{1}{4} - \frac{1}{2}\right)\right]$ (2) (2)

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ (2)